A Path Optimization Scheme for a Numerically Controlled Remote Manipulator

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During the current academic year the Case Engineering Design Center has been investigating the possibilities of applying numerical control techniques to a remote manipulator. This work has been sponsored by the Space Nuclear Propulsion Agency through NASA Grant Number NsG-728. The following is a brief resume of progress to date.

The initial step was to gain a working familiarity with remote manipulation. On November 4 and 5, Design Center personnel visited the Nuclear Rocket Disassembly station in Nevada and attended the Project Rose Seminar there. A graduate student was able to confer with engineers at Programmed and Remote Systems, a manipulator manufacturer, and to observe their facilities. These experiences and some preliminary study led to the choice of the PAR model 6000 as a basis for design, and a 1/2 scale un-powered model was constructed.

The details of building a path interpolation system based on an in-loop computer depend on proven techniques, and initial designs for the manipulator appeared to be quite straight-forward. However, it soon developed that a new approach must be incorporated to cope with the redundancies between the hand and the support structure of the manipulator. This required an optimizing algorithm, or program;

and the principal study has hence revolved on this algorithm. A scheme was developed and computer simulated by Mr. P. W. Hammond which minimizes the manipulator momentum for each point along its path. Mr. Hammond presented a paper describing his approach at the Symposium on Human Factors in Engineering on May 7, and a copy is appended hereto. Mr. J. T. Beckett has also developed a scheme based on minimum point-to-point time.

The principal conclusion drawn from these studies is that although very amenable to real-time computation, the incremental approach does not result in a satisfactory termination of the path. Present work is being directed toward applying some more comprehensive, although slow-computing feedback to the rapid incremental system, in hopes of obtaining the advantages of both.

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A Path Optimization Scheme for a Numerically Controlled Remote Manipulator

The objective of applying numerical control to a remote manipulator is to allow the operator to divert more of his attention from the control of the machine itself to the task at hand. The philosophy of our present research is that the operator need only specify the destination, or desired position for the hand of the manipulator; and the control computer will generate the required motions to reach that destination. This would appear to be very straight-forward.

However, in generating the motions of the manipulator, the computer encounters the following redundancy: To obtain versatility, practical manipulators usually have more degrees of freedom than the minimum required to reach every point in space. Thus merely specifying the exact path of the hand does not determine a unique sequence of positions for the manipulator, but only a family of sequences. From these, the computer must select one, and it is only reasonable to base that selection on some criterion of effectiveness. In other words, to optimize the motions of the manipulator, the computer must utilize a rule of decision, or algorithm.

The implementation, and even the definition of an optimum path is quite involved. Theoretically, the absolutely optimum path cannot be specified unless the entire future operation of the machine is known.

Even if we ignore posterity, specifying an optimum requires consideration of the consequences of any move on the remainder of the path. This presents a search problem of staggering complexity. In real time, when even a few seconds of delay between command and execution is intolerable; such an approach is eliminated.

Thus we are forced to accept the necessity of ignoring the consequences of a particular movement, and basing our optimization upon such immediately available factors as dynamics. This is undeniably a compromise, but not necessarily a practical handicap. It was decided as the first step in our study to evaluate such an approach.

As a starting point, the assumption was made that the position and orientation of the hand would move linearly from its initial to its final positions. This can be justified because the operator should be able to visualize the path to avoid obstacels. Furthermore, by subtracting final from initial values and dividing into the desired rate, the computer can easily break a straight line path into increments. This is essential to our optimizing algorithm.

The criterion of effectiveness which is the basis of this algorithm is that the path which has the minimum momentum stored in the manipulator at each point along its travel is the optimum path. Again, this type of criterion has the advantage that the path can be generated deterministically without advance calculation of the entire path, since the optimizing proceeds on a point-to-point basis.

To test the above criterion, a computer simulation of several typical paths has been run. A mathematical model of the PAR model 6000 remote manipulator was used for this purpose. The PAR 6000 is illustrated in Figure 1. This manipulator is suspended by a telescopic tube 20 feet long from a carriage that rides in a bridge 30 feet long. The bridge can in turn be traversed along rails. The manipulator can be pivoted about the axis of the tube, and has pivots with horizontal axes at the shoulder, elbow, and wrist. Thus the manipulator has seven degrees of freedom, while the position of the hand can be described by five coordinates (X, Y, Z, azimuth, and elevation).

From information in the manufacturer's brochure, the linear and angular inertias encountered at the various degrees of freedom are estimated as:

$$I_{Wrist} = 2 \text{ ft - slugs}$$

$$I_{Elbow} = 4.6 \text{ ft - slugs}$$

$$I_{Shoulder} = 7.6 \text{ ft - slugs}$$

$$I_{Azimuth} = 6.2 \sin^2 \theta_s + 4.2 \sin^2 \theta_E + 8.4 \sin \theta_s \sin \theta_E \text{ ft - slugs}$$

$$M_{Tube} = 21 \text{ slugs}$$

$$M_{Bridge} = 40 \text{ slugs}$$

$$M_{Traverse} = 51 \text{ slugs}$$

As would be expected, the angular intertia about the tube axis depends on the configuration of the arm. Because the other joint angles are all referenced to vertical, their inertias are independent.

The momentum stored in a particular degree of freedom is equal to the product of the inertia times the velocity at that degree of freedom. The criterion function F is just the sum of the magnitude of these momentums.

$$F = \left[I_w \frac{d\Phi_w}{dt}\right] + \left[I_E \frac{d\Phi_E}{dt}\right] + \left[I_s \frac{d\Phi_s}{dt}\right] + \left[I_{Azimuth}\right] \frac{d(\text{Azimuth})}{dt} +$$

$$\left[M_{Tube} \frac{d(\text{Tube})}{dt}\right] + \left[M_{Bridge} \frac{d(\text{Bridge})}{dt}\right] + \left[M_{Traverse} \frac{d(\text{Traverse})}{dt}\right]$$

In incremental terms where $\Delta t = 1$, this becomes:

$$F = \left[I_w \Delta \Phi_w\right] + \left[I_E \Delta \Phi_E\right] + \left[I_s \Delta \Phi_s\right] + \left[I_{Azimuth} \Delta(\text{Azimuth})\right] +$$

$$\left[M_{Tube} \Delta(\text{Tube})\right] + \left[M_{Bridge} \Delta(\text{Bridge})\right] + \left[M_{Traverse} \Delta(\text{Traverse})\right]$$

The coordinates of the hand of the manipulator are related to the machine configuration as follows:

θ = Azimuth

$\phi = \Phi_w$

$$X = \text{Traverse} + \cos(\text{Azimuth})\left[1.5 \sin(\Phi_s) + 1.5 \sin(\Phi_E) + 1.7 \sin(\Phi_w)\right]$$

$$Y = \text{Bridge} + \sin(\text{Azimuth})\left[1.5 \sin(\Phi_s) + 1.5 \sin(\Phi_E) + 1.7 \sin(\Phi_w)\right]$$

$$Z = \text{Tube} + 1.5 \cos(\Phi_s) + 1.5 \cos \Phi_E + 1.7 \cos(\Phi_w)$$

Since there are seven machine variables and only five hand coordinates, these equations cannot be immediately inverted. However, if two machine variables are arbitrarily chosen as independent, the remaining five can be expressed in terms of the hand coordinates and these two. Study of the above equations leads us to choose Φ_s and Φ_E, with the following results:

(1a) $\text{Azimuth} = \theta$

(1b) $\Phi_w = \phi$

(1c) $\text{Traverse} = X - \cos(\theta)\left[1.5 \sin(\Phi_s) + 1.5 \sin(\Phi_E) + 1.7 \sin(\phi)\right]$

(1d) $\text{Bridge} = Y - \sin(\theta)\left[1.5 \sin(\Phi_s) + 1.5 \sin(\Phi_E) + 1.7 \sin(\phi)\right]$

(1e) $\text{Tube} = Z - 1.5 \cos(\Phi_s) - 1.5 \cos(\Phi_E) - 1.7 \cos(\phi)$

For small increments, these become:

$$\Delta \text{Azimuth} = \Delta \theta$$

$$\Delta \Phi_w = \Delta \phi$$

$$\Delta \text{Traverse} = \Delta X + \Delta \theta \sin(\theta)\left[1.5 \sin(\Phi_s) + 1.5 \sin(\Phi_E) + 1.7 \sin(\phi)\right] - \cos\theta \left[1.5 \Delta \Phi_s \cos \Phi_s + 1.5 \Delta \Phi_E \cos(\Phi_E) + 1.7 \Delta \phi \cos(\phi)\right]$$

$$\Delta \text{Bridge} = \Delta Y - \Delta \theta \cos(\theta)\left[1.5 \sin(\Phi_s) + 1.5 \sin(\Phi_E) + 1.7 \sin(\phi)\right] - \sin\theta \left[1.5 \Delta \Phi_s \cos \Phi_s + 1.5 \Delta \Phi_E \cos(\Phi_E) + 1.7 \Delta \phi \cos(\phi)\right]$$

$$\Delta \text{Tube} = \Delta Z + 1.5 \Delta \Phi_s \sin(\Phi_s) + 1.5 \Delta \Phi_E \sin(\Phi_E) + 1.7 \Delta \phi \sin(\phi)$$

When these expressions are substituted into the criterion function, we see that the only incremental machine variables that appear are $\Delta \Phi_s$ and $\Delta \Phi_E$. Finding the minimum value of F is thus a two variable search problem on $\Delta \Phi_s$ and $\Delta \Phi_E$, if we ignore the variations of the machine variables during the search. This is possible for small $\Delta \Phi_E$ and $\Delta \Phi_s$ except for $I_{Azimuth}$, since fixing Φ_E and Φ_s would mask the interaction effect. We therefore re-calculate $I_{Azimuth}$ at the average position, namely at $\Phi_E + 1/2\, \Delta \Phi_E$

and $\Phi_s + 1/2 \Delta \Phi_s$.

$$I_{Azimuth} = 6.2 \left[\sin^2(\Phi_s) + \Delta \Phi_s \cos(\Phi_s) \sin(\Phi_s)\right]$$
$$+ 4.2 \left[\sin^2(\Phi_E) + \Delta\Phi_E \cos(\Phi_E) \sin(\Phi_E)\right]$$
$$+ 8.4 \sin(\Phi_s) \sin(\Phi_E) + 4.2 \left[\Delta \Phi_s \cos(\Phi_s) \sin(\Phi_E)\right.$$
$$\left. + \Delta \Phi_E \cos(\Phi_E) \sin(\Phi_s)\right]$$

We can now write the criterion function in terms of $\Delta\Phi_s$ and $\Delta\Phi_E$ in all its glory:

$$F(\Delta \Phi_s, \Delta \Phi_E) = |2 \Delta \phi| + |4.6 \Delta \Phi_E| + |7.6 \Delta \Phi_s| + |\{4.2 \cos(\Phi_E)$$

$$[\sin(\Phi_E) + \sin(\Phi_s)] \Delta \Phi_E + \cos(\Phi_s)[6.2 \sin(\Phi_s) + 4.2 \sin(\Phi_E)] \Delta \Phi_s +$$

$$[6.2 \sin^2(\Phi_s) + 4.2 \sin^2(\Phi_E) + 8.4 \sin(\Phi_s) \sin(\Phi_E)]\} \Delta \theta | \quad +$$

$$| 21 \{[1.5 \sin(\Phi_E)] \Delta \Phi_E + [1.5 \sin(\Phi_s)] \Delta \Phi_s + [\Delta Z +$$

$$1.7 \sin(\phi) \Delta \phi]\} | + | 40 \{-1.5 \cos(\Phi_E) \sin(\theta)) \Delta \Phi_E +$$

$$(-1.5 \cos(\Phi_s) \sin(\theta)) \Delta \Phi_s + [\Delta Y - \cos(\theta)(1.5 \sin(\Phi_s) +$$

$$1.5 \sin(\Phi_E) + 1.7 \sin(\phi)) \Delta \theta - 1.7 \sin(\theta) \cos(\theta) \Delta \phi]\} | +$$

$$\left| 51 \left\{ \left[-1.5 \cos (\Phi_E) \cos \theta \right] \Delta \Phi_E + \left[-1.5 \cos (\Phi_s) \cos \theta \right] \Delta \Phi_s + \right.\right.$$

$$\left[\Delta X - \sin (\theta) (1.5 \sin (\Phi_s) + 1.5 \sin (\Phi_E) + 1.7 \sin (\phi)) \Delta \theta - \right.$$

$$\left.\left. 1.7 \cos (\phi) \cos (\theta) \Delta \phi \right] \right\} \right|$$

If all variabled are held fixed except for $\Delta \Phi_E$ and $\Delta \Phi_s$, we see that this equation is of the form:

$$(3) \quad F(\Delta \Phi_E, \Delta \Phi_s) = \sum_{i=1}^{6} \left| a_i \Delta \Phi_E + b_i \Delta \Phi_s + c_i \right| + k$$

Each of the terms of the summation is a linear equation in two variables, and the plot of its magnitude is a plane surface reflecting off the $\Delta \Phi_E$, $\Delta \Phi_s$ plane at the line $a_i \Delta \Phi_E + b_i \Delta \Phi_s + c_i = 0$. The sum of such surfaces is also planar except over the six reflection lines, where there are ridges. It is easy to show that the minimum value of such a surface lies above one of the intersections of these reflection lines.

The minimum can only lie on one of the planar facets if that facet is level; in which case we may arbitrarily locate it at the edge, i.e., on a ridge.

The minimum can only lie on one of the ridges if that ridge is horizontal; in which case we may arbitrarily locate it at one end of the ridge, i.e., at a ridge intersection.

Hence the mimimum can always be found at a ridge intersection, or directly over a reflection line intersection. But six coplanar lines can intersect in at most $1/2\ (6)(6-1) = 15$ points. Thus rather than employing a statistical two variable search to find the minimum of F, we need only find the 15 intersections of the reflection lines, evaluate F at each point, and choose the smallest value. This is a tremendous advantage of our choice of criterion function. It should be stressed that the values chosen for $\Delta \phi_S$, and $\Delta \phi_E$ must be small enough to permit use of our equations. If this is not true then smaller path increments for the hand must be specified.

We can now describe in detail the process by which the minimum momentum path is generated. The desired path for the hand is specified by a start point (X, Y, Z, θ, ϕ) and a gradient $(\Delta X, \Delta Y, \Delta Z, \Delta \theta, \Delta \phi)$. In addition the initial values of ϕ_S and ϕ_E must be specified. The computer then advances through the following steps:

1. Using equations (1a - 1e), the values for the Tube, Bridge, Traverse, Azimuth, and ϕ_W are calculated.

2. With these values and the given increments ΔX, ΔY, ΔZ, $\Delta \theta$, and $\Delta \phi$ all the coefficients a_i, b_i, and c_i are evaluated in the equation (3) form of equation(2).

3. The intersection of a pair of reflection lines $a_i \Delta \phi_E + b_i \Delta \phi_s + c_i = 0$ and $a_j \Delta \phi_E + b_j \Delta \phi_s + c_j = 0$ is found through simultaneous solution, that is:

$$\Delta \phi_E = \frac{c_j b_i - c_i b_j}{a_i b_j - a_j b_i} \qquad \Delta \phi_s = \frac{c_i a_j - c_j a_i}{a_i b_j - a_j b_i}$$

4. The criterion function (2) is evaluated at this intersection point, using the coefficients calculated in step 2.

5. A running variable, F_{min}, is compared with this new value of F; and if F is smaller than F_{min} assumes the value of F. F_{min} is initially defined to be very large to insure that it assumes the first value calculated. On completion of this step the computer returns to step 3. Thus after all 15 values have been observed F_{min} will take on the smallest of them. Similar running variables identify the values of $\Delta \phi_E$ and $\Delta \phi_s$ that gave the best criterion.

6. The best values of $\Delta \xi_E$ and $\Delta \xi_s$ are added on to the previous values to obtain the "initial conditions" for the next step. The increments ΔX, ΔY, ΔZ, $\Delta \theta$, $\Delta \phi$ are also added to their previous values. Using these new values, the computer returns to step 1 and begins again.

7. A running variable counts up the number of times the above loop has been executed, and stops the process after a specified number has been reached.

This algorithm is illustrated in the flow chart of Figure 2.

Based on the algorithm described above, a program for the UNIVAC 1107 computer was written, using the ALGOL programming language. This program is given the initial start point and gradient for the hand, and the initial values of Φ_E and Φ_s; and calculates the positions of all the machine variables for 100 steps along the path. Several different sets of initial conditions were run with this program. Of these, four in particular were selected as typical of certain characteristics of the optimizing method, and are used to illustrate them here.

The first run is a check on the maximum permissible increment for which the equations are valid. It consists of four computations of the same path, broken into 5, 25, 100, and 500 increments. This showed that the largest increments give a considerable discrepancy in final configurations compared to the samllest increments, whereas the difference between the 100 and 500 increment path is slight. From these results we form a "rule-of-thumb" that the linear increments should never exceed .01 feet and the angular increments .01 radian.

The second run demonstrates the solution to an input that gives a moderately complicated path. The manipulator is asked to rotate its aximuth 180 degrees while traveling to the left and back from its initial position. The locii of the hand and shoulder as seen from above are shown in Figure 3.

The third and fourth runs illustrate two defficiencies of the optimizing method. These criticisms might be described in anthropomorphic terms as "lack of foresight" and "lack of imagination." To illustrate the former, the manipulator arm was started with the arm flexed and told to move forward. Because of the relatively low momentum involved in straightening the arm, that is the computer's first reaction. However, when the arm is fully straightened; the traverse system must then provide the motion. In practice, it might be more desirable to save the more dexterous arm movements for the final positioning.

The "lack of imagination" is illustrated by the final run, in which the arm is straight and horizontal, and told to move away from the direction it is pointing. Clearly it would be best to flex the arm to obtain this motion, bringing the hand and shoulder together. However, because the initial flexing of the arm hardly moves the hand at all, the computer rejects this possibility, and simply moves the traverse system. It is not capable of making a strategic sacrifice to obtain future advantages.

This last deficiency is by far the most serious and is the inherent defect in any point-by-point trajectory control system. Instead of optimizing the entire path at once, the computer is actually sequentially optimizing many small concantenated paths. It is an interesting, but unfortunate paradox that the sum of many optimum paths is not itself optimum. Figure 4 shows one solution by an imaginative computer (namely, a human being) to the same initial conditions as Figure 3. Bearing in mind that the bridge, traverse, and tube comprise 90 °/o of the manipulator weight; it would clearly seem desirable to minimize the motion of these parts by a short shoulder path. On this basis, the path of Figure 4 is clearly superior to Figure 3. However, to obtain this advantage required, a shoulder path longer than Figure 3 during the first two intervals--even though it is always shorter thereafter. It was this initial sacrifice that eluded the computer.

Our conclusions, then, are that the point-to-point optimization approach becomes less and less satisfactory as the length and complexity of the path increases. However, the speed with which it can be generated and its short-path performance lead us to believe that a modified scheme, in which data from a slow comprehensive path evaluation guides and corrects the fasten point-to-point method; will generate quite acceptable paths. We consider the synthesis and testing of such a hybrid approach the next step in our study.